9.4.6 Trade-off method for utility functions

All previously described methods for determining utility functions suffer from the same issue; they assume that the decision maker's answers to questions about certainty equivalents to simple lotteries etc. can be interpreted in a sense that is in line with expected utility. In Chapter 13, we will see that this assumption is critical and that decision makers usually give distorted answers even if only simple lotteries are involved. At this point, however, we would like to stress a problem that is important to all utility elicitation methods discussed so far. When making intuitive decisions (like when answering questions for certainty equivalents to simple lotteries), small probabilities have a larger impact on the evaluation of a lottery than expected utility theory prescribes. Other probabilities are systematically over- or under-weighted as well. Neglecting these distortions results in systematically distorted utility functions. Bleichrodt et al. (2001) analyze how one can adjust answers that suffer from systematically distorted probabilities and hence elicit undistorted utility functions. The exact approach, however, is beyond the scope of this book and we refer the reader to the original paper.

An alternative is to apply methods which do not suffer from these biases. The trade-off method (which has nothing in common with the trade-off method from Chapter 6 except for the name) by Wakker and Deneffe (1996) is one of these methods. Compared with previous methods, it is more complicated. This is however more than offset as distorted probabilities no longer result in a distorted utility function.

The basic idea of the trade-off method for eliciting utility functions is similar to the method of equal utility differences. The decision maker produces a sequence of consequences which all have the same utility difference. While the method of equal utility differences breaks down if the probability $p = 0.5$ is systematically distorted (e.g. is treated like a probability of 0.4), the trade-off method is immune to such a bias.

The trade-off method requires two consequences x_a and x_b with $x_b \succ x_a$ to be chosen such that they – in the best case – lie outside of the interval $[x_{min}, x_{max}]$ of relevant consequences (the two consequences x_a and x_b are only needed for comparison reasons; we assume they are less than x_{min}).

Again, we set $x_0=x_{min}$ and the decision maker is asked for a consequence x_1 that makes him indifferent between the lotteries $(x_0, p; x_b, 1–p)$ and $(x_1, p; x_a, 1–p)$. Presumably, a probability of $p = 0.5$ will make this question particularly easy to answer. However, the trade-off method works with every other p. Some algebra shows that this indifference statement implies $u(x_1) – u(x_0) = (1–p)/p\cdot(u(x_b)–u(x_a))$. The utility difference between x_1 and x_0 is given by the value $(1–p)/p\cdot(u(x_b)–u(x_a))$. This value is unknown and furthermore it is affected by the potentially distorted probabilies p and $(1–p)$. However, it is not necessary to know this value of the utility difference because the next step delivers a similar insight. We ask the decision maker for a consequence x_2 that makes him indifferent between $(x_1, p; x_b, 1–p)$ and $(x_2, p; x_a, 1–p)$. From this indifference, we deduce that the utility difference between x_2 and x_1 is also given by the same value $u(x_2) - u(x_1) = (1-p)/p \cdot (u(x_b) - u(x_a))$. In particular we conclude that $u(x_2) - u(x_1) = u(x_1) - u(x_0)$ and this holds true independent of the exact choice of the potentially distorted probability p. The next steps are obvious now: we produce a whole series of indifference statements $(x_i, p; x_b, 1-p) \sim (x_{i+1}, p; x_a, 1-p)$ which all result in equal utility indifferences. A convenient choice of x_a and x_b (the closer they are, the smaller the distances in the sequence of consequences become) makes it possible for us to reach (or exceed) the consequence x_{max} within four to five steps, as in the method of equal utility differences. The normalization of the utility function is then as usual.

We want to illustrate the trade-off method with an example. Assume a decision maker wants to know his utility function on the interval $[x_{min} = €1{,}000,\ x_{max} = €10{,}000]$. We set $x_a = €100$ and $x_b = €500$, so that these two consequences lie outside of the previously specified interval. The next steps are shown in Figure 9-18. In a first step, we ask for a consequence x_1 that makes the decision maker indifferent between the lotteries (€1,000, 30%; €500, 70%) and (x_1, 30%; €100, 70%). As already noted, we expect a probability of $p = 0.5$ to ease the whole procedure for the decision maker. For didactical reasons, however, we here chose $p = 0.3$. Assume the decision maker states $x_1 = €2{,}500$. Then, in a second step, we ask for x_2 that makes him indifferent between the lotteries (€2,500, 30%; €500, 70%) and (x_2, 30%; €100, 70%). We obtain a further value, say, $x_2 = €6{,}000$ that we can use for the next step. Assume the decision maker reveals $x_3 = €11{,}000$; we then stop the elicitation process as €11,000 exceeds $x_{max} = €10{,}000$.

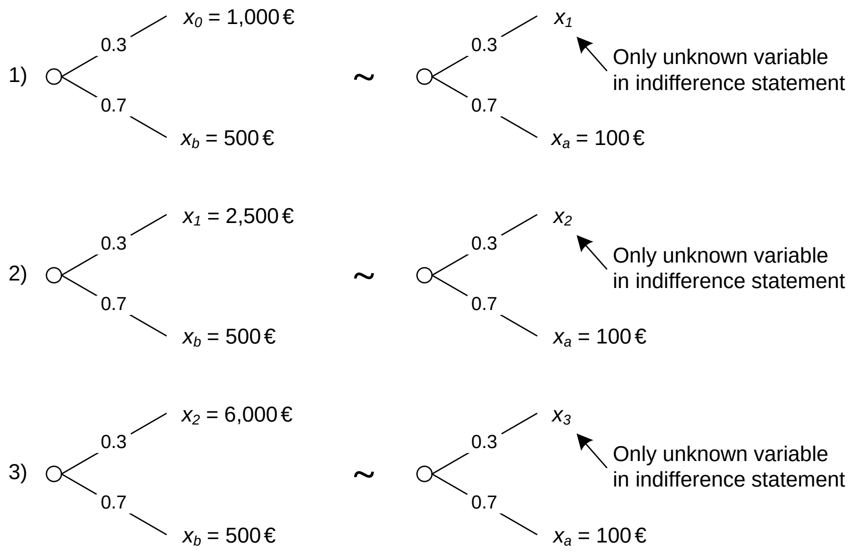

Figure 9-18: Lotteries and indifference statements in the trade-off method

The resulting points on the decision maker's utility function are shown in Figure 9-19. To facilitate the understanding, we have plotted two utility axes: on the left hand side, a non-normalized, arbitrarily scaled utility axis and on the right hand side a utility axis normalized on the interval [x_{min} = €1,000, x_3 = €11,000]. You can see that, on the non-normalized utility axis, the scaled utility difference $(1–p)/p·[u(x_b)–u(x_a)]$ resulting from the utility difference $u(x_b)–u(x_a)$ and our choice of the lottery's probability p = 0.3 determine the equal utility difference between all three elicited utility values $u(x_1)$, $u(x_2)$ and $u(x_3)$. The well-known normalization then leads to the utility values $u(x_1)$ = 1/3, $u(x_2)$ = 2/3 and $u(x_3)$ = 1 as is depicted on the normalized utility axis on the right hand side.

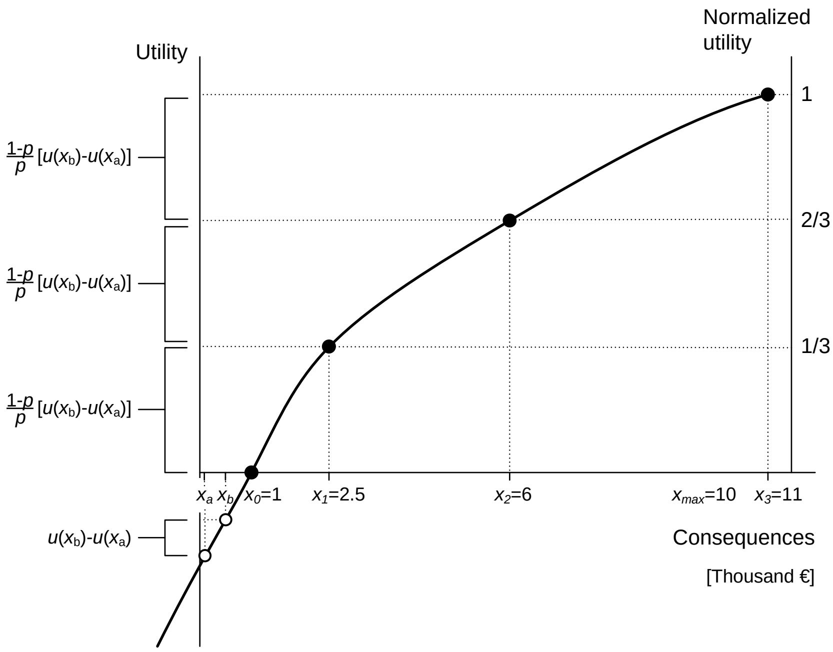

Figure 9-19: Utility function elicited with the trade-off method